TECHNISCHE MECHANIK II
Formelsammlung

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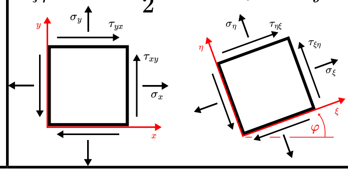
Spannungen

Druck-Zug-Spannung	$\sigma = \frac{F}{A}$	Räumlicher Spannungszustand	$\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$ $\tau_{ij} = \tau_{ji}$
Torsionsspannung	$\tau = \frac{M_T}{W_T} = \frac{M_T}{I_T} r$	Ebener Spannungszustand	$\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix}$
Schubspannung	$\tau_z = \frac{Q_z \cdot S_y^*}{I_y \cdot t(z)}$ $S_y^* = \int z \, dA$	Hauptspannungen	$\sigma_{1,2} = \frac{\sigma_y + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + \tau_{zy}^2}$ $\tan 2\varphi^* = \frac{2\tau_{zy}}{\sigma_y - \sigma_z}$
Biegespannung	$\sigma_z = \frac{M_z}{W_z} = \frac{M_z}{I_z} y$ $\sigma_y = \frac{M_y}{W_y} = \frac{M_y}{I_y} z$ $M_z = N_x \cdot e_y$ $M_y = N_x \cdot e_z$	Transformation	$\sigma_\xi = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\varphi + \tau_{xy} \sin 2\varphi$ $\sigma_\eta = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\varphi - \tau_{xy} \sin 2\varphi$ $\tau_{\xi\eta} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\varphi + \tau_{xy} \cos 2\varphi$ 

Dehnung/Verzerrung

Dehnung	$\varepsilon = \frac{\Delta l}{l_0}$ $\varepsilon = \frac{\partial u}{\partial x} \quad \varepsilon = \frac{\partial v}{\partial y} \quad \varepsilon = \frac{\partial w}{\partial z}$	DGL der Verschiebung DGL der Biegelinie	$EA \cdot u' = N(x) + (\alpha \cdot \Delta T_x)'$ $EIw'' = -M(x)$ $EIw''' = -Q(x)$ $EIw'''' = q(x)$
Verschiebung	$u = \Delta l$	Längenänderung	$\Delta l = \frac{F \cdot l_0}{E \cdot A}$
Schubmodul Elastizitätsmodul Poissonzahl	$G = \frac{E}{2(1+\nu)} \quad G = \frac{\tau}{tan\alpha}$ $E = \frac{\sigma}{\varepsilon}$ $\nu = -\frac{\Delta d/d}{\Delta l/l}$	Verzerrungstensor	$\varepsilon = \begin{pmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{pmatrix}$ $\gamma_{xy} = \gamma_{yx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ $\gamma_{yz} = \gamma_{zy} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$ $\gamma_{zx} = \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$
Transformation	$\varepsilon_\xi = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\varphi + \gamma_{xy} \sin 2\varphi$ $\varepsilon_\eta = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\varphi - \gamma_{xy} \sin 2\varphi$ $\gamma_{\xi\eta} = -\frac{\varepsilon_x - \varepsilon_y}{2} \sin 2\varphi + \gamma_{xy} \cos 2\varphi$		

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Flächenträgheitsmoment

Flächenträgheitsmoment (Axial)	$I_y^* = I_y + z_s^2 \cdot A$ $I_z^* = I_z + y_s^2 \cdot A$	Flächenträgheitsmoment (Polar)	$I_p = I_y + I_z = \int r^2 dA$
Flächenträgheitsmoment (Biaxial)	$I_{yz}^* = I_{yz} - y_s z_s \cdot A$	Widerstandsmoment	$W_y = \frac{I_y}{e_z}$ $\quad W_z = \frac{I_z}{e_y}$ $e_z\ ,\ e_y = Abstand\ bis\ Schwerpunkt$
Haupt-Flächenträgheitsmoment	$I_{1/2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$	$\tan(2\alpha^*) = \frac{2I_{yz}}{(I_y - I_z)}$	
Transformation	$I_\eta = \frac{I_y + I_z}{2} + \frac{I_y - I_z}{2}\cos 2\varphi + I_{yz}\sin 2\varphi$ $I_\xi = \frac{I_y + I_z}{2} - \frac{I_y - I_z}{2}\cos 2\varphi - I_{yz}\sin 2\varphi$	$I_{\xi\eta} = -\frac{I_y - I_z}{2}\sin 2\varphi + I_{yz}\cos 2\varphi$	

Doppelbiegung

Schiefe Biege und Normalspannung	$\sigma_{(y,z)} = \frac{N}{A} + \frac{(M_y \cdot I_z - M_z \cdot I_{yz}) \cdot z - (M_z \cdot I_y - M_y \cdot I_{yz}) \cdot y}{I_y \cdot I_z - (I_{yz})^2}$ $I_{yz} = 0\ \ (Wenn\ das\ System\ in\ einer\ Achse\ symmetrisch\ ist)$		
Spannungsnulllinie Koordinaten	$y_0 = \frac{N \cdot I_z}{A \cdot M_z}$ $z_0 = -\frac{N \cdot I_y}{A \cdot M_y}$	Neigung der Spannungsnulllinie	$tan(\beta_0) = \frac{M_z \cdot I_y}{I_z \cdot M_y}$ $Winkel\ ab\ positiver\ y\!-\!Achse\ (Gegenuhrzeigersinn)$
Vergleichsspannungen Normalspannungshypothese (NSH): Rankine	$\sigma_v = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$		
Schubspannungshypothese (SSH): Tresca	$\sigma_v = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$		
Gestaltänderungs-energiehypothese (GEH): Von Mises	$\sigma_v = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$		

Schubspannung infolge Querkraft und Torsion

Schubspannung infolge Querkraft	$\tau_z = \frac{Q_z \cdot S_y^*}{I_y \cdot t}$	Statisches Moment (Erstes Flächenmoment)	$S_y^* = A^* \cdot z^*$ $S_z^* = A^* \cdot y^*$
Schubspannung infolge Torsion (Geschlossene Querschnitte)	$\tau = \frac{M_T}{2 \cdot t \cdot A_m}$	Torsionsträgheitsmoment (Geschlossene Querschnitte)	$I_T = \frac{4A_m^2}{\oint \frac{ds}{t}}$ $\oint \frac{ds}{t} = \frac{l_1}{t_1} + \frac{l_2}{t_2} + \ldots$
Schubspannung infolge Torsion (Offene Querschnitte)	$\tau = \frac{M_T}{I_T} t$	Torsionsträgheitsmoment (Offene Querschnitte)	$I_T = \beta \sum \frac{1}{3} h \cdot t^3$
Verdrehwinkel	$\varphi = \frac{M_T \cdot l}{G \cdot I_T}$	Scherwinkel / Verdrillung	$\vartheta = \frac{\varphi}{l}$

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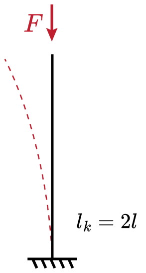
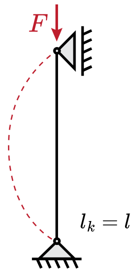
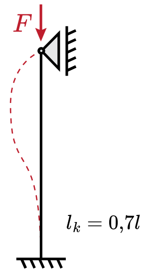
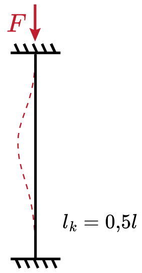
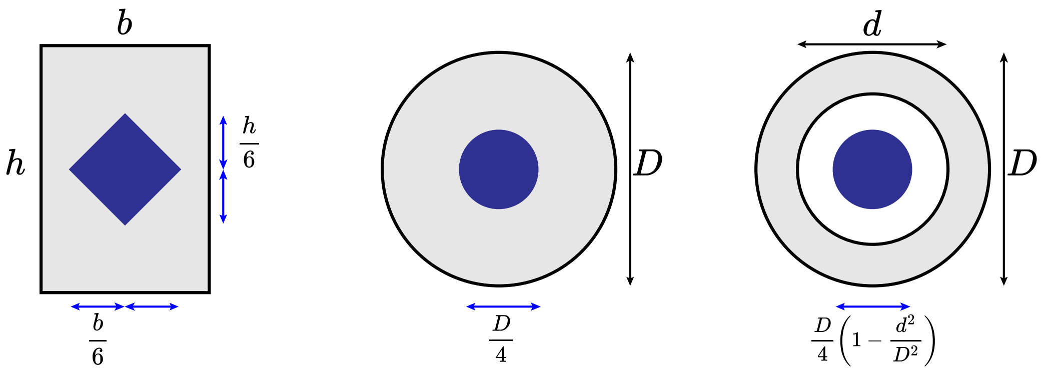
Knickung

Euler'sche Knicklast	$F_{krit} = \dfrac{\pi^2 EI}{\ell_k^2}$	DGL der Knickung	$w'''' + \lambda^2 w'' = 0$ $\lambda^2 = \dfrac{F}{EI}$ $w = A\cos\lambda x + B\sin\lambda x + C\lambda x + D$
Euler Fall 1. 	Euler Fall 2. 	Euler Fall 3. 	Euler Fall 4. 

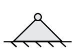
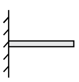
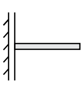
Kernfläche

Kernpunkte	$y_k = -\dfrac{i_z^2}{y_0}$ $\quad z_k = -\dfrac{i_y^2}{z_0}$	Trägheitsradius	$i_y = \sqrt{\dfrac{I_y}{A}}$ $\quad i_z = \sqrt{\dfrac{I_z}{A}}$

y_0 ; z_0 *Asbtand zwischen Schwerpunkt des Systems und Nulllinie*



Randbedingungen

Lager	Geometrische Randbedingung		Statische Randbedingung	
Fest-/Loslager 	$w = 0$	$w' \neq 0$	$M = 0$	$Q \neq 0$
Einspannung 	$w = 0$	$w' = 0$	$M \neq 0$	$Q \neq 0$
Freies Ende 	$w \neq 0$	$w' \neq 0$	$M = 0$	$Q = 0$
Parallelführung 	$w \neq 0$	$w' = 0$	$M \neq 0$	$Q = 0$

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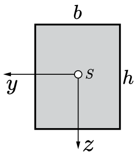
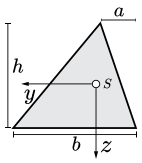
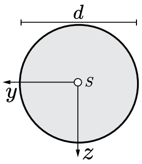
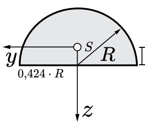
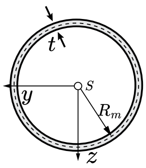
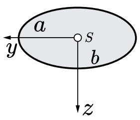
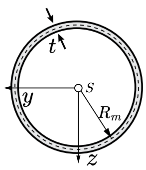
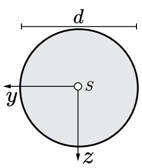
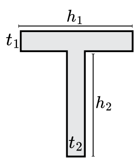
Flächenträgheitsmoment Tabelle

Querschnitt	A	I_y	I_z	I_{yz}	W_y	W_z
	$A = bh$	$I_y = \frac{bh^3}{12}$	$I_z = \frac{hb^3}{12}$	$I_{yz} = 0$	$W_y = \frac{bh^2}{6}$	$W_z = \frac{hb^2}{6}$
	$A = \frac{1}{2}bh$	$I_y = \frac{bh^3}{36}$	$I_z = \frac{bh}{36}(b^2 - ba + a^2)$	$I_{yz} = \frac{bh^2}{72}\cdot(2a - b)$	$W_y = \frac{I_y \cdot 2}{b}$	$W_z = \frac{I_z \cdot 2}{h}$
	$A = \frac{\pi d^2}{4}$	$I_y = \frac{\pi d^4}{64}$	$I_z = \frac{\pi d^4}{64}$	$I_{yz} = 0$	$W_y = \frac{\pi d^3}{32}$	$W_z = \frac{\pi d^3}{32}$
	$A = \frac{\pi r^2}{2}$	$I_y = 0,110 \cdot r^4$	$I_z = \frac{\pi}{8}r^4$	$I_{yz} = 0$	$W_o = 0{,}191r^3$ $W_u = 0{,}259r^3$	$W_z = \frac{\pi}{8}r^3$
	$A = 2\pi R_m t$	$I_z = \pi R_m^3 t$	$I_z = \pi R_m^3 t$	$I_{yz} = 0$	$W_y = \pi R_m^2 t$	$W_z = \pi R_m^2 t$
	$A = \pi ab$	$I_y = \frac{\pi}{4}ab^3$	$I_z = \frac{\pi}{4}ba^3$	$I_{yz} = 0$	$W_y = \frac{\pi}{4}ab^2$	$W_z = \frac{\pi}{4}ba^2$

Torsionsträgheitsmoment Tabelle

Querschnitt	I_T	W_T
	$I_T = 2\pi R_m^3 t$	$W_T = 2\pi R_m^2 t$
	$I_T = \frac{\pi R^4}{2}$	$W_T = \frac{\pi R^3}{2}$
	$I_T = \beta \sum \frac{1}{3} h \cdot t^3$	$W_T = \frac{1}{3}\frac{\sum ht^3}{t_{max}}$

Profilform